GLENWOOD

Student No. | | | | | | | | | |

Student Name: _____

Teacher Name: _____

2020

YEAR 12
TRIAL
HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided
- For questions in Section II, show relevant mathematical reasoning and/or calculations

Total marks: 100

Section I – 10 marks – Pages 2 - 5
- Attempt Questions 1-10
- Allow about 15 minutes for this section

Section II – 90 marks – Pages 6 - 32
- Attempt Questions 11 - 34
- Allow about 2 hours and 45 minutes for this section

SECTION 1

10 MARKS
Attempt questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for questions 1-10

1 If $f(x) = x^2 - 2x$, what is the value of $f'(3)$?

(A) 0

(B) 6

(C) 3

(D) 4

2 What amount does an investment of \$20 000 grow to after 3 years at 5% p.a. compounded quarterly?

(A) \$20 759.41

(B) \$23 152.50

(C) \$23 215.09

(D) \$23 223.68

3 A discrete probability distribution is shown in the table below.

$X = x$	1	2	3	4	5
$P(X = x)$	0.23	0.38	0.21	0.13	0.05

The expected value is

(A) 1.00

(B) 1.56

(C) 2.39

(D) 5.71

4 Simplify $\dfrac{x^2+5x+6}{x^2-9}$

(A) $\dfrac{x+2}{x+3}$

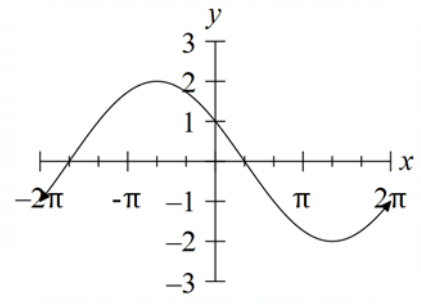
(B) $\dfrac{x+2}{x-3}$

(C) $\dfrac{x+3}{x-3}$

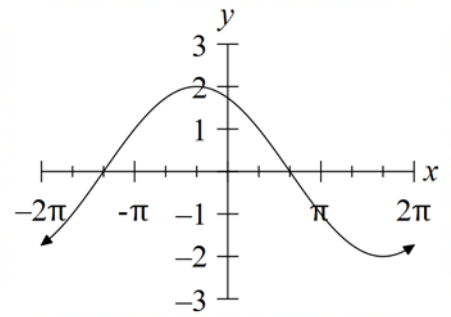
(D) $\dfrac{x-2}{x-3}$

5 Which of the following best represents the graph of $g(x) = 2\cos\left(\frac{x}{2}+\frac{\pi}{3}\right)$?

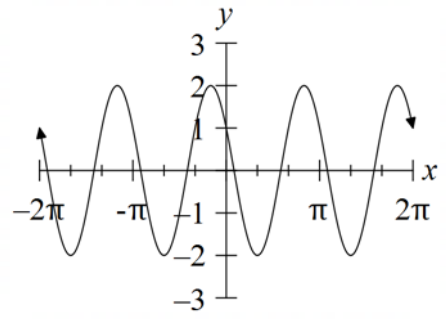
(A)



(B)



(C)



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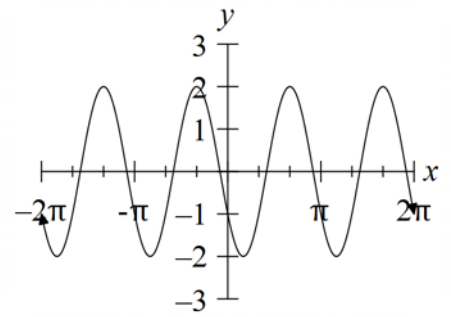


6 A set of data is normally distributed with a mean of 8.6 and a standard deviation of 0.7. The percentage of scores that lie between 8.6 and 9.3 is:

(A) 17.5 %

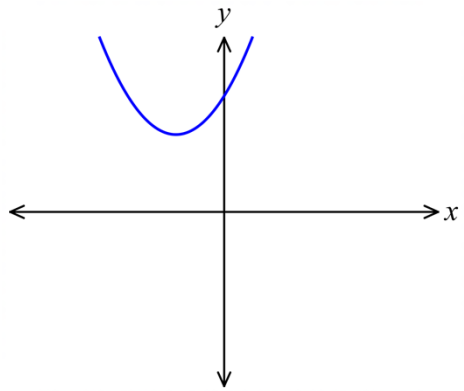
(B) 34 %

(C) 68 %

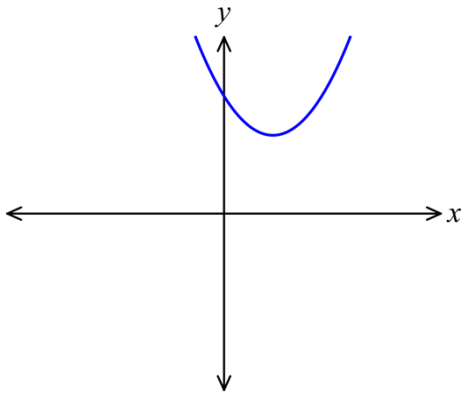
(D) 95 %

7 Which diagram best shows the graph of the parabola $y = 2 - (x + 1)^2$?

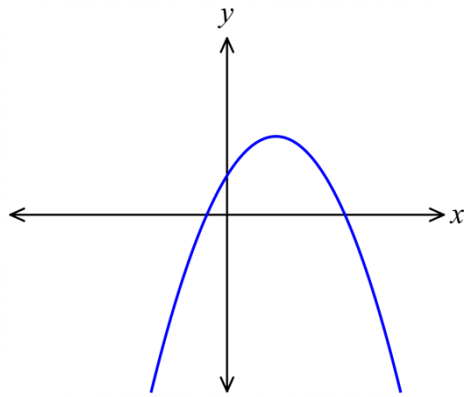
(A)



(B)



(C)



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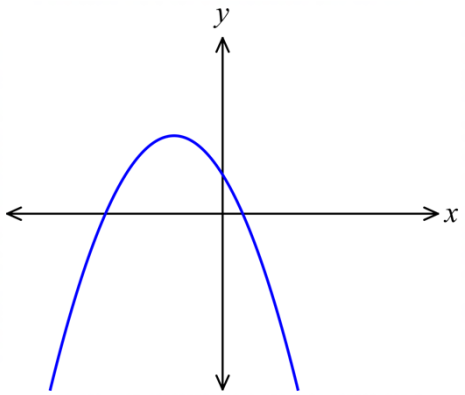


8 Evaluate $\int_0^2 (3x^4 + 2x)dx$

(A) $\frac{96}{5}$

(B) $\frac{116}{5}$

(C) $\frac{136}{5}$

(D) 100

9 Carol bought a triangular block of land with sides of length 9, 5 and 10 metres respectively. What is the area of her land in m^2 ?

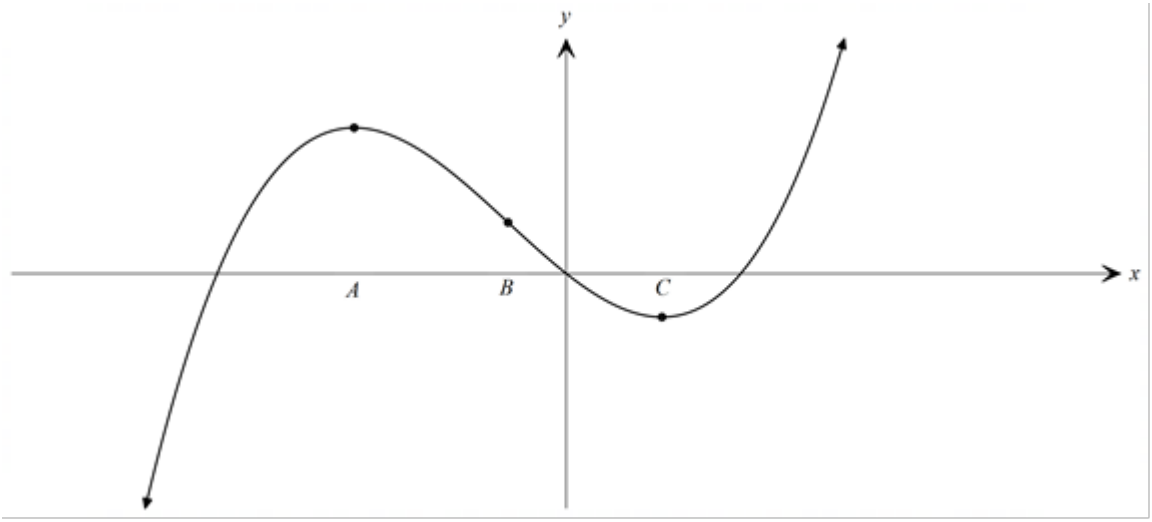
(A) $22.5\ sin\ 86°11'$

(B) $20\ sin\ 106°36'$

(C) $14\ sin\ 86°11'$

(D) $45\ sin\ 106°36'$

10 The graph of $y = f(x)$ is shown below.
$x = A$ and $x = C$ are stationary points, and $x = B$ is a point of inflection.



Over what domain is $f'(x) < 0$ and $f"(x) > 0$?

(A) $(-\infty, A)$

(B) (A, B)

(C) (B, C)

(D) (C, ∞)

2020 TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Student name

Mathematics Advanced

Student Number

Section II Answer Booklet

Teacher Name

90 marks
Attempt Questions 11 – 34
Allow about 2 hour and 45 minutes for this section

Instructions

- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of responses.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the back of the booklet. If you use this space, clearly indicate which question you are answering.

Question 11 (2 marks)

Express $\frac{8}{3-\sqrt{5}}$ in the form $a + b\sqrt{5}$ **2**

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Question 12 (2 marks)

Given $\cos\theta = \frac{2}{7}$ and $270° \leq \theta \leq 360°$, find the exact value of $\sin\theta$. **2**

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Question 13 (3 marks)

Consider the function $f(x) = \sqrt{x}$ and $g(x) = 49 - x^2$ **3**

Find $f\big(g(x)\big)$ and state its domain and range.

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Question 14 (2 marks)

The function $y = (x-2)^3 - 2$ is translated 5 units to the right and 4 units up. **2**

Find the equation of the transformed equation.

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Question 15 (3 marks)

(a) Show that $\frac{4x-3}{x} = \frac{-3}{x} + 4$ **1**

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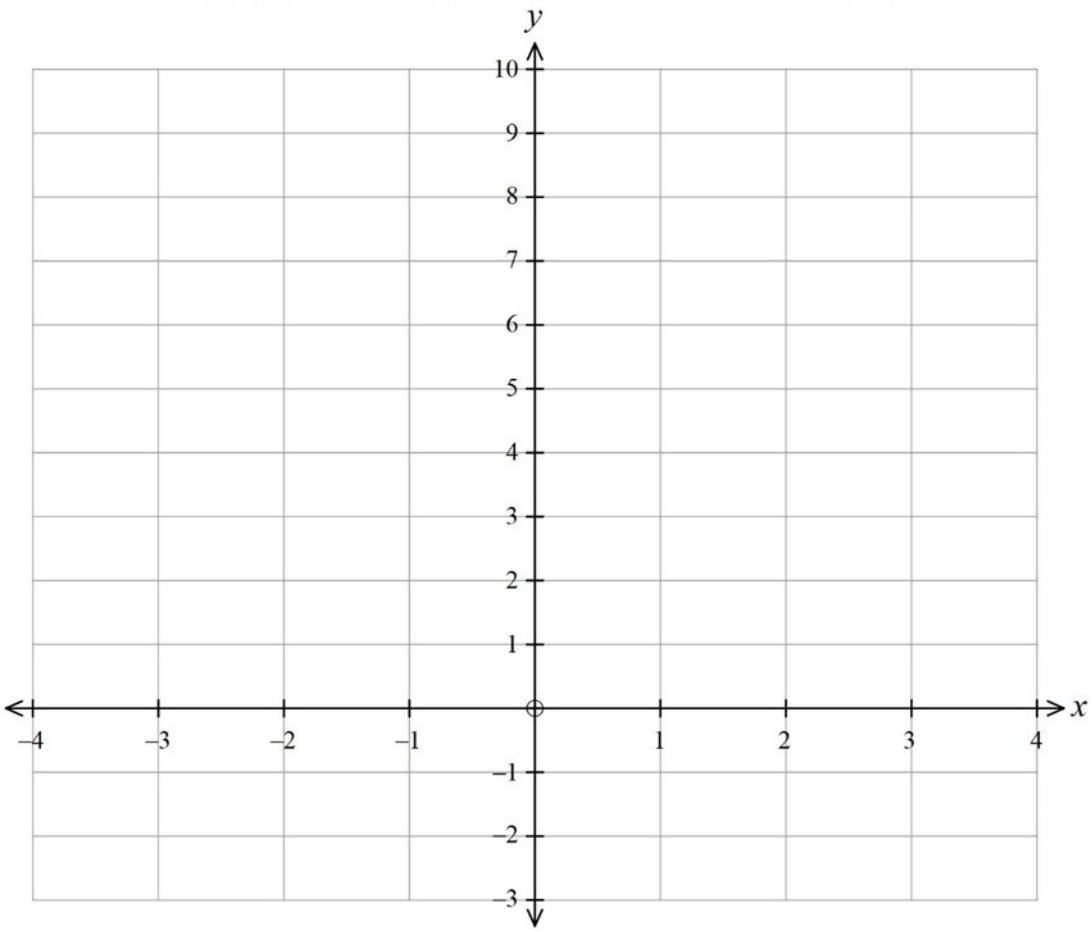
(b) Hence or otherwise, sketch the graph of $y = \frac{4x-3}{x}$ showing any asymptotes and the x- intercept. **2**



Question 16 (4 marks)

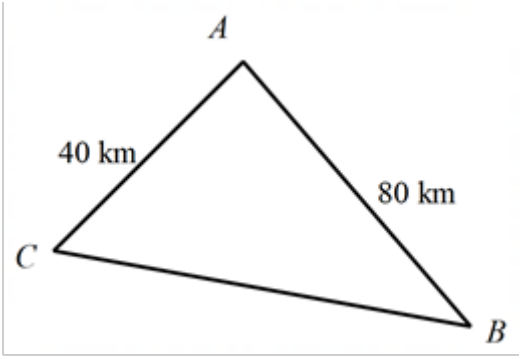
(a) Show that the derivative of $\ln\left(\frac{3+x}{3-x}\right)$ is $\frac{6}{9-x^2}$ **2**

(b) Hence or otherwise find $\int \frac{1}{9-x^2}\,dx$. **2**

Question 17 (4 marks)

Three towns, *A*, *B*, and *C* form a triangle.

Town *A* is 80 km from Town *B* and Town *C* is 40 km from Town *A* as shown below:



The bearing of Town *B* from Town *A* is 130°. The bearing of Town *C* from Town *A* is 240°.

(a) Find the area of the triangle formed by the three towns, to the nearest square kilometre. **2**

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(b) Using the cosine rule, find the distance between Town *B* and Town *C*, to the nearest kilometre. **2**

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Question 18 (4 marks)

A curve with the equation $y = f(x)$, has $\frac{dy}{dx} = x^3 + 2x - 7$.

(a) The point $P(2,4)$ lies on the curve. Find y in terms of x. **2**

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(b) Find and equation for the normal to the curve at P, in the form $ax + by + c = 0$, where a, b and c are integers. **2**

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Question 19 (3 marks)

Solve $2\cos\left[2\left(x-\frac{\pi}{6}\right)\right] = 1$ in the domain $[0, 2\pi]$. **3**

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Question 20 (3 marks)

Find the median of the continuous probability distribution defined as $f(x) = \frac{x^3}{64}$ in the domain $[0, 4]$. **3**

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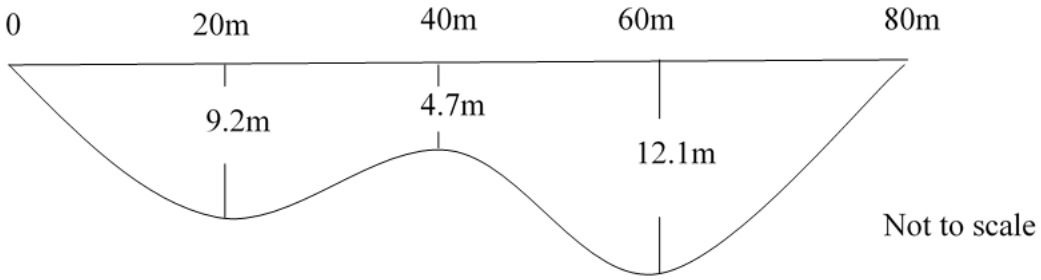
Question 21 (4 marks)

The following diagram represents a cross-section through a river. The depth of the river is marked every 20 metres.



(a) Use the trapezoidal's rule to estimate the area of the cross-section. **2**

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(b) If the river is flowing at the rate of 6 m/s, what is the volume of water that passes through the cross-section each hour ? **2**

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Question 22 (2 marks)

Find $\int \frac{2x+2}{4x^2+8x+1}dx$. **2**

Question 23 (3 marks)

The table below shows the present value interest factors for some monthly interest rates and loan periods in months.

Present value of $1				
Period	0.0060	0.0065	0.0070	0.0075
46	40.09350	39.64965	39.21263	38.78231
47	40.84841	40.38714	39.93310	39.48617
48	41.59882	41.11986	40.64856	40.18478
49	42.34475	41.84785	41.35905	40.87820

(a) Find the present value, if $3200 is contributed per month for 46 months at 0.75% per month. Answer to the nearest cent. **1**

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(b) Annabelle borrows $27 000 for a car. She arranges to repay the loan with monthly repayments over 4 years. She is charged 7.8% per annum interest. Find Annabelle's monthly repayment. Answer to the nearest cent. **2**

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Question 24 (5 marks)

A uniform continuous random variable with probability density function is shown below.

$$f(x) = \begin{cases} ax^2, & 0 \leq x \leq 6 \\ 0, & otherwise \end{cases}$$

(a) Find the value of *a.* **2**

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(b) Find $P(3 \leq X \leq 5)$ **3**

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Questions 11-24 are worth 44 marks in total

Question 25 (5 marks)

(a) Show that the graphs of $y = \sin x$ and $y = \cos x$ in the domain $0 \leq x \leq 2\pi$. intersect at $x = \frac{\pi}{4}, \frac{5\pi}{4}$ **2**

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(b) Find the area bounded by $y = \sin x$ and $y = \cos x$ from $x = \frac{\pi}{4}$ to $x = \frac{5\pi}{4}$, in exact values. **3**

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Question 26 (2 marks)

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Find $\frac{d}{dx}\left(x^3 e^{x^3}\right)$.

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Question 27 (2 marks) **2**

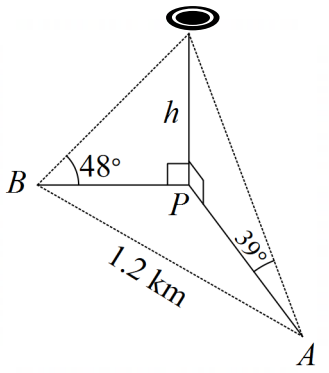
The formula for first principles differentiation is

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Use this formula to find the derivative of $f(x) = x^2 + 3x$.

Question 28 (6 marks)

Anthony sees an unidentified flying object in the sky hovering above point P in a direction of N23°W and at an angle of elevation of 39°. He calls his brother Benjamin, who sees the same object in a direction of S83°E and at an angle of elevation of 48°. According to their GPS, Anthony and Benjamin are 1.2 km away from each other.



(a) Find $\angle APB$ **2**

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(b) Show that $AP = h \cot 39°$. **1**

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(c) Use the cosine rule to find h, the height of the object above point P, to the nearest 10 metres. **3**

Question 29 (5 marks)

The population of an ant colony is modelled by the equation $P = 1000e^{kt}$, where t is measured in weeks.

(a) Find the initial population. **1**

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(b) After 5 weeks the population is 15 000. **2**

Show that $k = 0.5416$, correct to 4 decimal places.

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(c) During which week will the population exceed 50 000 for the first time ? **2**

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Question 30 (6 marks)

Max did a survey of a group of people he knew about their age and how much they earn each week. The results are shown in the table below.

Age (years) (x)	18	45	28	15	32	68
Wage (\$/week) (W)	715	2350	1530	438	1690	1320

(a) Using your calculator find (r), the correlation coefficient, and explain what type and strength of correlation this data gives. **2**

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(b) Using your calculator, find the equation of the least-squares regression line in the form **1**

$W = Bx + A$ where A and B are integers.

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(c) Use your equation to estimate the earnings of a 50 year-old worker. **1**

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Question 30 continues next page

(d) Could your equation from Part (b) be used to make valid estimates for ages greater than 68 and less than 15 years? **2**

Validate your response with calculations and or reasons.

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Question 31 (6 marks)

A particle moves in a straight line. At time t seconds, its distance x metres from a fixed point 0 on the line is given by $x = 1 - \cos 2t$.

(a) Sketch the graph of x as a function of t for $0 \leq t \leq \pi$. **2**

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(b) Using your graph, or otherwise, to find the times when the particle is at rest and the position of the particle at these times. **2**

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Question 31 continues next page

(c) Find the velocity of the particle when $t = \frac{\pi}{4}$. **1**

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(d) When is the particle's velocity greater than 1 m/s? **1**

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Question 32 (6 marks)

A function is given by $f(x) = x^4 - 4x^3$.

(a) Find the stationary points, determine their nature, and find any points of inflection. **4**

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Question 32 continues next page

(b) Sketch the graph of $f(x) = x^4 - 4x^3$, labelling the stationary points, inflection points and axis intercepts. **2**

Question 33 (3 marks)

In a laundry basket there are shirts in three sizes, 3 are small, 8 are medium and 5 are large.

Ed takes two shirts from the basket at random.

(a) Draw a tree diagram, clearly showing the outcomes and the correct probabilities on each branch. **2**

(b) Find the probability that Ed chooses two shirts of the same size. **1**

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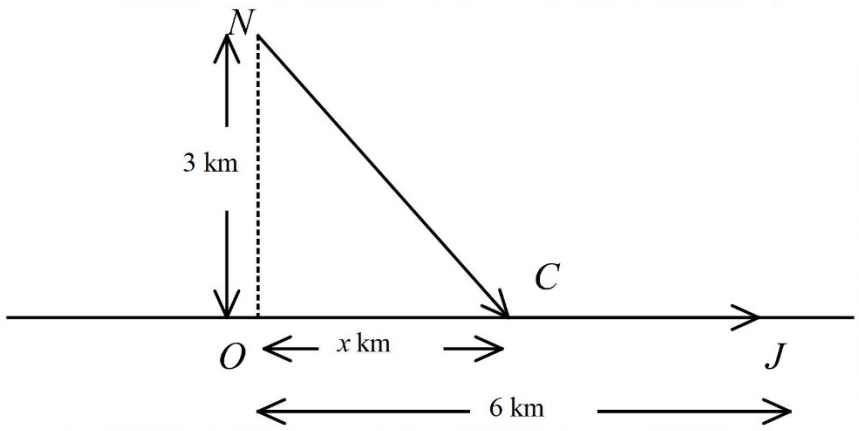
Question 34 (5 marks)

Nathan is on a paddle board in the ocean 3 km from the nearest point *O* on a straight beach. Nathan needs to meet his friend Jarrod who is located 6 km along the beach from the point *O*. Nathan paddles at a rate of 4 km/h to a point C on the beach and then walks at a rate of 5 km/h along the beach to Jarrod. **5**



Show that the total time it takes Nathan to reach Jarrod is given by:

$T(x) = \dfrac{\sqrt{x^2+9}}{4} + \dfrac{6-x}{5}$.

Hence, find the minimum time it will take Nathan to reach Jarrod.

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Answer space for Question 34 continues next page

END OF EXAMINATION

Section II Extra writing space

If you use this space, clearly indicate which question you are answering.

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GLENWOOD

Student No. | | | | | | | | | |

Student Name: Final ~~Draft~~

Teacher Name: Solution

2020

YEAR 12
TRIAL
HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
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Total marks: 100

Section I – 10 marks – Pages 2 - 5
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SECTION 1

10 MARKS
Attempt questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for questions 1-10

1 If $f(x) = x^2 - 2x$, what is the value of $f'(3)$?

(A) 0
(B) 6
(C) 3
(D) 4

D

$f'(x) = 2x - 2$.
$f'(3) = 2(3) - 2$
$= 4$.

2 What amount does an investment of \$20 000 grow to after 3 years at 5% p.a. compounded quarterly?

(A) \$20 759.41
(B) \$23 152.50
(C) \$23 215.09
(D) \$23 223.68

C

$A = 20000\left(1 + \frac{0.05}{4}\right)^{3\times4}$
$=$

3 A discrete probability distribution is shown in the table below.

$X = x$	1	2	3	4	5
$P(X = x)$	0.23	0.38	0.21	0.13	0.05

The expected value is

(A) 1.00
(B) 1.56
(C) 2.39
(D) 5.71

C

$E(x) = \sum x p(x)$
$=$

4 Simplify $\frac{x^2+5x+6}{x^2-9}$

$\frac{(x+2)(x+3)}{(x-3)(x+3)}$

(A) $\frac{x+2}{x+3}$

(B) $\frac{x+2}{x-3}$

B

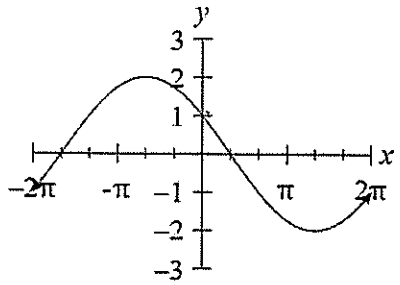
(C) $\frac{x+3}{x-3}$

(D) $\frac{x-2}{x-3}$

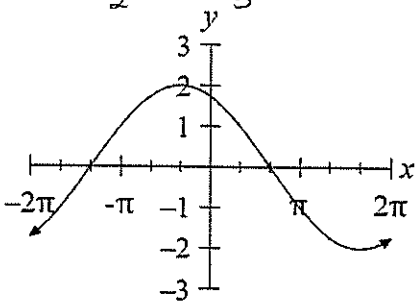
5 Which of the following best represents the graph of $g(x) = 2\cos\left(\frac{x}{2}+\frac{\pi}{3}\right)$? $2\cos \pi/3$

(A)

A



(B) $2\cos\frac{1}{2}\left(x+\frac{2\pi}{3}\right)$



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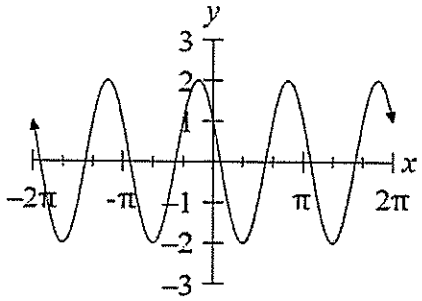

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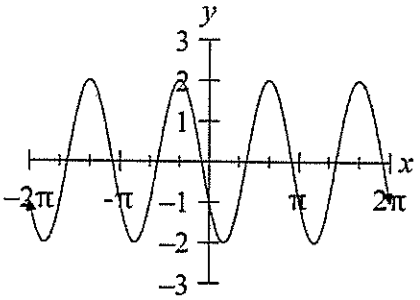


6 A set of data is normally distributed with a mean of 8.6 and a standard deviation of 0.7. The percentage of scores that lie between 8.6 and 9.3 is:

B

(A) 17.5 %

(B) 34 %

(C) 68 %

(D) 95 %

$\mu = 8.6$

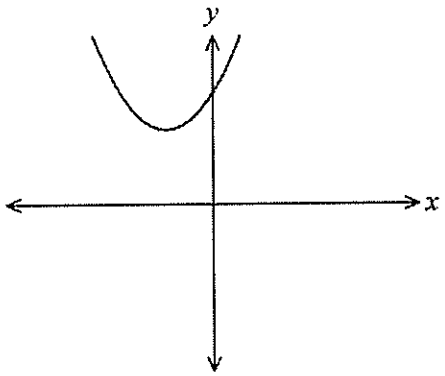
$\sigma = 0.7$

$z = \frac{x-\mu}{\sigma} = \frac{9.3-8.6}{0.7} = 1$

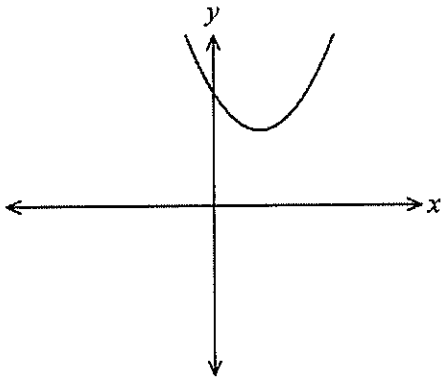
7 Which diagram best shows the graph of the parabola $y = 2 - (x + 1)^2$?

D

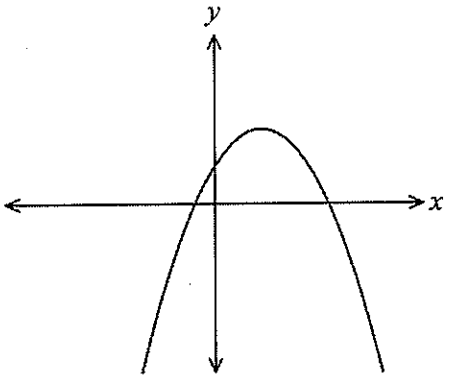
(A)



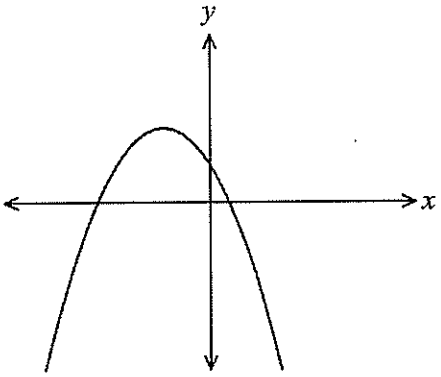
(B)



(C)



(D)



8 Evaluate $\int_0^2 (3x^4 + 2x)dx$

B

(A) $\frac{96}{5}$

(B) $\frac{116}{5}$

(C) $\frac{136}{5}$

(D) 100

~~12x~~ $\left[\frac{3x^5}{5} + \frac{2x^2}{2}\right]_0^2$

$= \left(\frac{96}{5} + 4\right) - 0$

$= \frac{116}{5}$

9 Carol bought a triangular block of land with sides of length 9, 5 and 10 *metres* respectively.

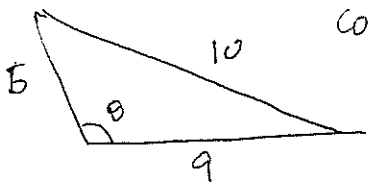
What is the area of her land in m^2 ?

(A) $22.5\ sin\ 86°11'$

(B) $20\ sin\ 106°36'$

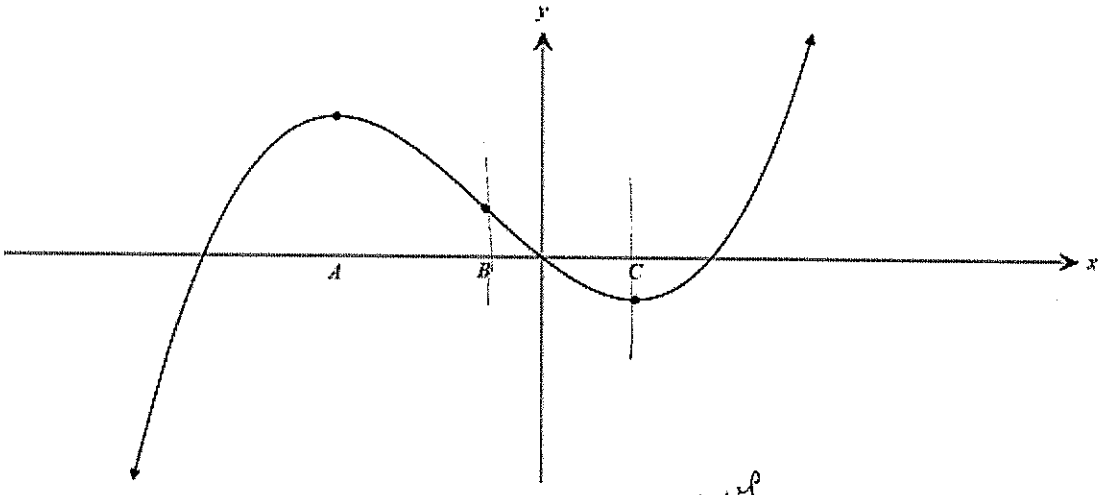
(C) $14\ sin\ 86°11'$

(D) $45\ sin\ 106°36'$



10 The graph of $y = f(x)$ is shown below.

$x = A$ and $x = C$ are stationary points, and $x = B$ is a point of inflection.



Over what domain is $f'(x) < 0$ and $f''(x) > 0$?

(A) $(-\infty, A)$

(B) (A, B)

(C) (B, C)

(D) (C, ∞)

2020 TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Student name

Mathematics Advanced

Student Number

Section II Answer Booklet

Teacher Name

90 marks 41 34
Attempt Questions 10 – 32
Allow about 2 hour and 45 minutes for this section

Instructions

- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of responses.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the back of the booklet. If you use this space, clearly indicate which question you are answering.

Question 11 (2 marks)

Express $\frac{8}{3-\sqrt{5}}$ in the form $a + b\sqrt{5}$ **2**

$$\frac{8}{3-\sqrt{5}} \times \frac{(3+\sqrt{5})}{(3+\sqrt{5})}$$

$$= \frac{8(3+\sqrt{5})}{9-5}$$

$$= \frac{8(3+\sqrt{5})}{4}$$

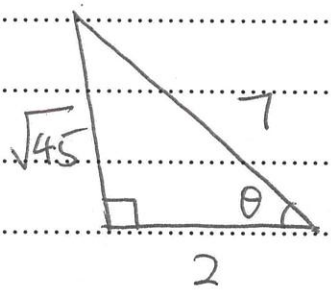
$$= 2(3+\sqrt{5})$$

$$= 6+2\sqrt{5}$$

Comments: Overall, well done
Some students got up to $\frac{8(3+\sqrt{5})}{4}$ but were unable to simplify correctly

Question 12 (2 marks)

Given $\cos\theta = \frac{2}{7}$ and $270° \leq \theta \leq 360°$, find the exact value of $\sin\theta$. **2**



In 4th quadrant, $\sin\theta$ is negative

$$\therefore \sin\theta = -\frac{\sqrt{45}}{7} = -\frac{3\sqrt{5}}{7}$$

Yes ✓
$\sqrt{45}$ ✓
7

Comments: many struggled/ left blank.
Some forgot to read the domain (4th quadrant)

Question 13 (3 marks)

Consider the function $f(x) = \sqrt{x}$ and $g(x) = 49 - x^2$ **3**

Find $f(g(x))$ and state its domain and range.

$f(g(x)) = \sqrt{49-x^2}$ (1)

Domain : $-7 \leq x \leq 7$, $[-7, 7]$ (1)

Range : $0 \leq y \leq 7$, $[0, 7]$ (1)

wrong brackets −1

Most students found $f(g(x)) = \sqrt{49-x^2}$

Of those, mainly were able to identify the correct domain/range

Question 14 (2 marks)

The function $y = (x-2)^3 - 2$ is translated 5 units to the right and 4 units up. **2**

Find the equation of the transformed equation.

$y = (x-2-5)^3 - 2 + 4$

$y = (x-7)^3 + 2$

(1) (1)

Comment:

Many wrote $y = (x+3)^3 + 2$.

Question 15 (3 marks)

(a) Show that $\frac{4x-3}{x} = \frac{-3}{x} + 4$ **1**

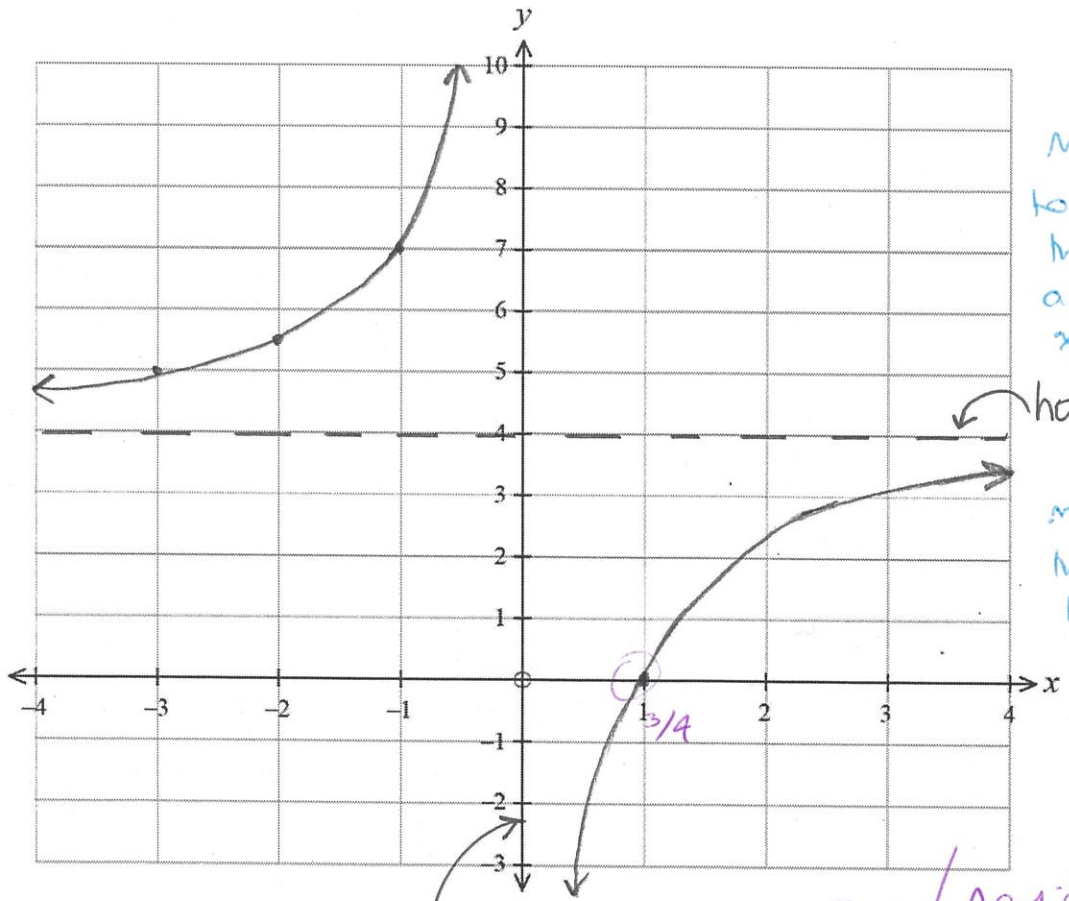
$\frac{4x-3}{x} = \frac{4x}{x} - \frac{3}{x}$

$= 4 - \frac{3}{x}$

Overall, well done.

$= \frac{-3}{x} + 4$

(b) Hence or otherwise, sketch the graph of $y = \frac{4x-3}{x}$ showing any asymptotes and the x- intercept. **2**



May forgot to label the horizontal asymptote and/or x-int.

horizontal asymptote $y=4$

Most identified the shape as a hyperbola.

vertical asymptote $x=0$

x-int/Asy: 1

curve 1

Question 16 (4 marks)

(a) Show that the derivative of $\ln\left(\frac{3+x}{3-x}\right)$ is $\frac{6}{9-x^2}$ **2**

$y = \ln\left(\frac{3+x}{3-x}\right)$

$y = \ln(3+x) - \ln(3-x)$

$y' = \frac{1}{3+x} - \left(-\frac{1}{3-x}\right)$

$= \frac{(3-x) + (3+x)}{9-x^2}$

$y' = \frac{6}{9-x^2}$

Many students did not realise $\ln\left(\frac{3+x}{3-x}\right) = \ln(3+x) - \ln(3-x)$

These students found $\frac{6}{(3-x)^2}$ $\frac{d}{dx}\left(\frac{3+x}{3-x}\right)$ but

① Quotient rule often and using $\frac{g(x)}{f(x)}$ did not complete the question after that step. but many only.

(b) Hence or otherwise find $\int \frac{1}{9-x^2}\,dx$. **2**

$\int \frac{1}{9-x^2}\,dx$

$= \frac{1}{6}\int \frac{6}{9-x^2}\,dx$

from part a

$= \frac{1}{6}\ln\left(\frac{3+x}{3-x}\right) + C$ No marks dedicated

Many students wrote

$\int \frac{1}{9-x^2}\,dx = \frac{-1}{2x}\int \frac{-2x}{9-x^2}\,dx$

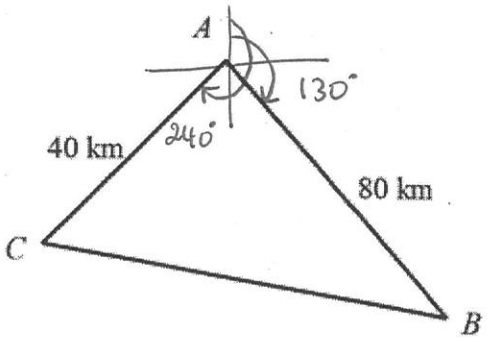
$= \frac{-1}{2x}\ln(9-x^2) + C$

Some forgot the +C.

Question 17 (4 marks)

Three towns, A, B, and C form a triangle.

Town A is 80 km from Town B and Town C is 40 km from Town A as shown below:



The bearing of Town B from Town A is 130°. The bearing of Town C from Town A is 240°.

(a) Find the area of the triangle formed by the three towns, to the nearest square kilometre. **2**

$\angle CAB = 240° - 130° = 110°$ (1)

$A = \frac{1}{2} \times 40 \times 80 \times \sin 110°$

$= 1503.508\ldots \text{ km}^2$ Overall, well done

$\approx 1504 \text{ km}^2$ (1)

(b) Using the cosine rule, find the distance between Town B and Town C, to the nearest kilometre. **2**

$BC^2 = 40^2 + 80^2 - 2 \times 40 \times 80 \cos 110°$

$= 10188.92892$

$BC = 100.9402245$ Overall, well done

$BC = 101$ km

Question 18 (4 marks)

A curve with the equation $y = f(x)$, has $\frac{dy}{dx} = x^3 + 2x - 7$.

(a) The point $P(2,4)$ lies on the curve. Find y in terms of x. **2**

$y = \int x^3 + 2x - 7 \, dx$

$y = \frac{x^4}{4} + x^2 - 7x + C$

Since $P(2,4)$ lies on the curve,

$4 = \frac{2^4}{4} + 2^2 - 7(2) + C$

$C = 10$

$\therefore y = \frac{x^4}{4} + x^2 - 7x + 10$

(b) Find and equation for the normal to the curve at P, in the form $ax + by + c = 0$, where a, b and c are integers. **2**

Gradient of the tangent at $P(2,4)$

$m = \frac{dy}{dx} = 2^3 + 2(2) - 7$

$= 5$

Gradient of normal is $-\frac{1}{5}$

Equation of normal:

$y - 4 = -\frac{1}{5}(x - 2)$

$5y - 20 = -x + 2$

$x + 5y - 22 = 0$

Question 19 (3 marks)

Solve $2\cos\left[2\left(x-\frac{\pi}{6}\right)\right] = 1$ in the domain $[0, 2\pi]$. 3

$\cos\left[2\left(x-\frac{\pi}{6}\right)\right] = \frac{1}{2}$ $\quad -\frac{\pi}{3} \leq 2x - \frac{\pi}{3} \leq \frac{11\pi}{3}$

$2(x - \pi/6) = -\pi/3, \pi/3, 5\pi/3, 7\pi/3, 11\pi/3$

$x - \pi/6 = -\pi/6, \pi/6, 5\pi/6, 7\pi/6, 11\pi/6$

$x = 0, \pi/3, \pi, 4\pi/3, 2\pi$

Question 20 (3 marks)

Find the median of the continuous probability distribution defined as $f(x) = \frac{x^3}{64}$ in the domain $[0, 4]$. 3

For median $\int_0^x \frac{x^3}{64}\,dx = 0.5$

$\frac{1}{64}\left[\frac{x^4}{4}\right]_0^x = 0.5$

$\frac{x^4}{256} = 0.5$

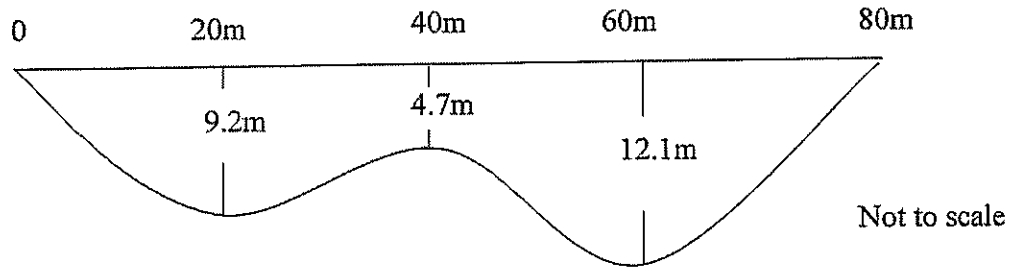
$x^4 = 128$

$x = 3.4$

Median is 3.4

Question 21 (4 marks)

The following diagram represents a cross-section through a river. The depth of the river is marked every 20 metres.



Not to scale

(a) Use the trapezoidal's rule to estimate the area of the cross-section. **2**

Area = $\frac{20}{2}\{0+0+2(9.2+4.7+12.1)\}$

$= 520\ m^2$

(b) If the river is flowing at the rate of 6 m/s, what is the volume of water that passes through the cross-section each hour ? **2**

Water rate = 6m/s

= 21600m/h

Volume of water each hour

$= 21600 \times 520$

$= 11\,232\,000\ m^3$

Question 22 (2 marks)

Find $\int \frac{2x+2}{4x^2+8x+1}dx$. **2**

using $\int \frac{f'(x)}{f(x)}dx = \ln|f(x)| + C$

$\int \frac{2x+2}{4x^2+8x+1}dx$ $\quad f(x) = 4x^2+8x+1$

$\quad f'(x) = 8x+8$

$= \frac{1}{4}\int \frac{4(2x+2)}{4x^2+8x+1}dx$

$= \frac{1}{4}\ln|4x^2+8x+1| + C$

Question 23 (3 marks)

The table below shows the present value interest factors for some monthly interest rates and loan periods in months.

Present value of $1				
Period	0.0060	0.0065	0.0070	0.0075
46	40.09350	39.64965	39.21263	38.78231
47	40.84841	40.38714	39.93310	39.48617
48	41.59882	41.11986	40.64856	40.18478
49	42.34475	41.84785	41.35905	40.87820

(a) Find the present value, if $3200 is contributed per month for 46 months at 0.75% per month. Answer to the nearest cent. **1**

, Period 46 months, interest 0.0075 per month

From table, Factor = 38.78231

$PV = 3200 \times 38.78231$

$= \$124\,103.39$

(b) Annabelle borrows $27 000 for a car. She arranges to repay the loan with monthly repayments over 4 years. She is charged 7.8% per annum interest. Find Annabelle's monthly repayment. Answer to the nearest cent. **2**

$r = \frac{0.078}{12} = 0.0065$ per month

$n = 4 \times 12 = 48$ months

Factor from table = 41.11986

Monthly repayment $= \frac{27\,000}{41.11986}$

$= \$656.62$

Question 24 (5 marks)

A uniform continuous random variable with probability density function is shown below.

$$f(x) = \begin{cases} ax^2, & 0 \le x \le 6 \\ 0, & otherwise \end{cases}$$

(a) Find the value of a. **2**

$\int_0^6 f(x) = 1$

$\int_0^6 ax^2 = 1$

$\left[\frac{ax^3}{3}\right]_0^6 = 1$

$72a = 1$

$a = \frac{1}{72}$.

(b) Find $P(3 \le X \le 5)$ **3**

$\int_3^5 f(x)\,dx$

$= \int_3^5 \frac{x^2}{72}\,dx$

$= \frac{1}{72}\left[\frac{x^3}{3}\right]_3^5$

$= \frac{1}{216}(5^3 - 3^3)$

$= \frac{49}{108}$

Questions 11-24 are worth 44 marks in total

Question 25 (5 marks)

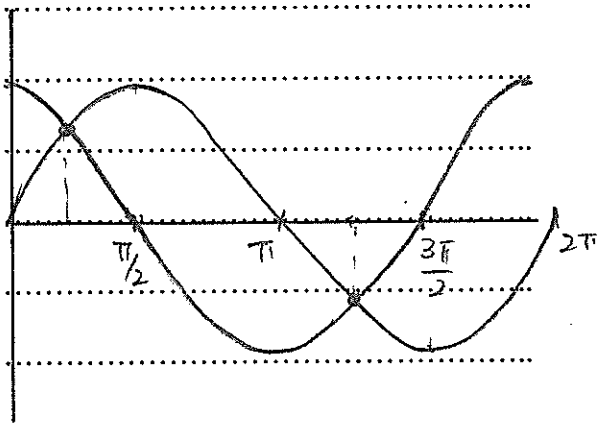
(a) Show that the graphs of $y = \sin x$ and $y = \cos x$ in the domain $0 \leq x \leq 2\pi$, intersect at $x = \frac{\pi}{4}, \frac{5\pi}{4}$ **2**

$\sin x = \cos x$

$\frac{\sin x}{\cos x} = 1$

$\tan x = 1$

$x = \frac{\pi}{4}, \frac{5\pi}{4}$



(b) Find the area bounded by $y = \sin x$ and $y = \cos x$ from $x = \frac{\pi}{4}$ to $x = \frac{5\pi}{4}$, in exact values. **3**

$$\int_{\pi/4}^{5\pi/4} \sin x - \cos x \, dx$$

$$= \left[-\cos x - \sin x\right]_{\pi/4}^{5\pi/4}$$

$$= \left[-\cos\left(\frac{5\pi}{4}\right) - \sin\left(\frac{5\pi}{4}\right)\right] - \left[-\cos\frac{\pi}{4} - \sin\frac{\pi}{4}\right]$$

$$= \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) - \left(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right)$$

$$= \frac{2}{\sqrt{2}} + \frac{2}{\sqrt{2}}$$

$$= \frac{4}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = 2\sqrt{2} \text{ units}^2$$

Question 26 (2 marks)

2

Find $\frac{d}{dx}\left(x^3 e^{x^3}\right)$.

$u = x^3$

$= 3x^2 e^{x^3} + x^3 . 3x^2 e^{x^3}$ $\quad u' = 3x^2$

$v = e^{x^3}$

$= 3x^2 e^{x^3} + 3x^5 e^{x^3}$ $\quad v' = 3x^2 e^{x^3}$

$= 3x^2 e^{x^3}(1 + x^3)$

Question 27 (2 marks) **2**

The formula for first principles differentiation is

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Use this formula to find the derivative of $f(x) = x^2 + 3x$.

$f(x+h) = (x+h)^2 + 3(x+h)$

$f'(x) = \lim_{h \to 0} \frac{(x+h)^2 + 3(x+h) - x^2 - 3x}{h}$

$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 + 3x + 3h - x^2 - 3x}{h}$

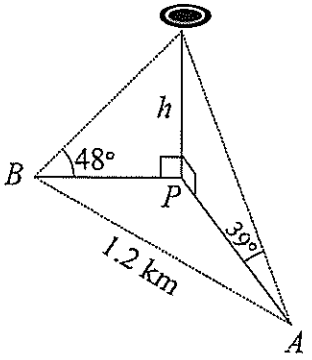
$= \lim_{h \to 0} \frac{h(2x + h + 3)}{h}$

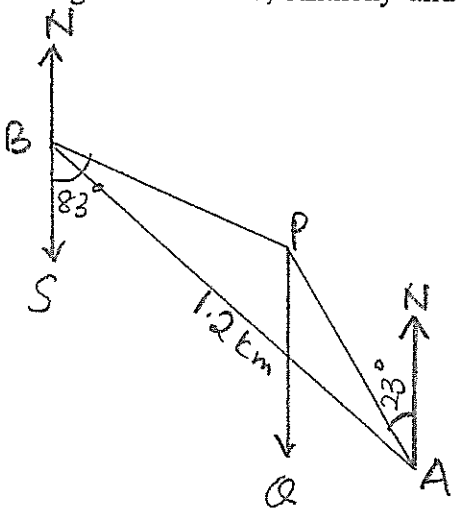
$= \lim_{h \to 0} 2x + h + 3$

$f'(x) = 2x + 3$

Question 28 (6 marks)

Anthony sees an unidentified flying object in the sky hovering above point P in a direction of N23°W and at an angle of elevation of 39°. He calls his brother Benjamin, who sees the same object in a direction of S83°E and at an angle of elevation of 48°. According to their GPS, Anthony and Benjamin are 1.2 km away from each other.





(a) Find $\angle APB$ **2**

$\angle BPQ = 180° - 83°$

$= 97°$ (cointerior)

$\angle QPA = 23°$ (alternate)

$\therefore \angle APB = 97° + 23°$

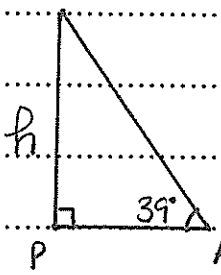
$= 120°$

(b) Show that $AP = h \cot 39°$. **1**

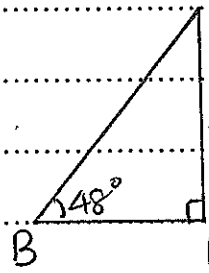
$\tan 39° = \frac{h}{AP}$

$AP = \frac{h}{\tan 39°}$

$= h \cot 39°$



(c) Use the cosine rule to find h, the height of the object above point P, to the nearest 10 metres. **3**



$\tan 48° = \frac{h}{BP}$

$BP = h \cot 48°$

Using cosine rule

$AB^2 = AP^2 + BP^2 - 2(AP)(BP)\cos 120°$

$1.2^2 = (h \cot 39°)^2 + (h \cot 48°)^2 - 2h\cot 39° h\cot 48° \cos 120°$

$1.2^2 = h^2\cot^2 39° + h^2\cot^2 48° - 2h^2 \cot 39° \cot 48° \cos 120°$

$= h^2(\cot^2 39° + \cot^2 48° - 2\cot 39° \cot 48° \cos 120°)$

$$h = \sqrt{\frac{1.2^2}{\cot^2 39° + \cot^2 48° - 2\cot 39° \cot 48° \cos 120°}}$$

$$= \sqrt{\frac{1.2^2}{\frac{1}{\tan^2 39°} + \frac{1}{\tan^2 48°} - 2\left(\frac{1}{\tan 39°}\right)\left(\frac{1}{\tan 48°}\right)\cos 120°}}$$

$= 0.646$ km

$h = 650$ m

Question 29 (5 marks)

The population of an ant colony is modelled by the equation $P = 1000e^{kt}$, where t is measured in weeks.

(a) Find the initial population. **1**

at $t = 0$

$P = 1000e^{0}$

$= 1000$

(b) After 5 weeks the population is 15 000. **2**

Show that $k = 0.5416$, correct to 4 decimal places.

at $t = 5$, $P = 15\,000$

$15000 = 1000e^{5k}$

$15 = e^{5k}$

$\ln 15 = \ln e^{5k}$

$\ln 15 = 5k$

$k = \frac{\ln 15}{5}$

$= 0.5416$

(c) During which week will the population exceed 50 000 for the first time ? **2**

$P = 1000e^{0.5416t}$ want $P > 50\,000$

$1000e^{0.5416t} = 50\,000$

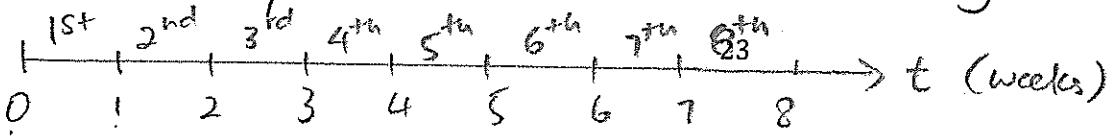
$e^{0.5416t} = 50$

$0.5416t = \ln 50$

$t = \frac{\ln 50}{0.5416}$

$t = 7.223\ldots$

∴ Population exceeds 50 000 during the 8th week.



Question 30 (6 marks)

Max did a survey of a group of people he knew about their age and how much they earn each week. The results are shown in the table below.

Age (years) (x)	18	45	28	15	32	68
Wage (\$/week) (W)	715	2350	1530	438	1690	1320

(a) Using your calculator find (r), the correlation coefficient, and explain what type and strength of correlation this data gives. **2**

r = 0.5263

positive, moderate correlation

(b) Using your calculator, find the equation of the least-squares regression line in the form **1**

$W = Bx + A$ where A and B are integers.

From the calculator, A = 706, B = 18

W = 18x + 706

(c) Use your equation to estimate the earnings of a 50 year-old worker. **1**

x = 50

W = 18 × 50 + 706

= $1606

Question 30 continues next page

(d) Could your equation from Part (b) be used to make valid estimates for ages greater than 68 and less than 15 years? No **2**

Validate your response with calculations and or reasons.

Different reasons possible. Some possibilities are:

- Since it is only a moderate correlation, it is not a valid way to estimate for younger or older people.

- Equation does not take into account that people below 15 and above 68 are not working.

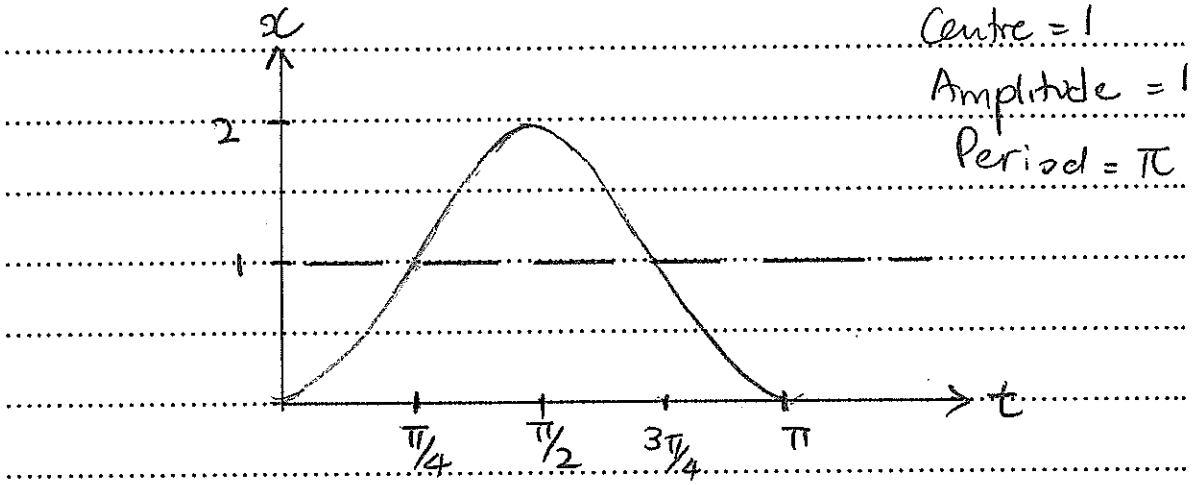
- Valid for interpolation between 15 and 45 years but is also affected by the outlier

- If the 68 year old's data was left out, the correlation coefficient, $r = 0.989$, which means a stronger correlation for 15 to 45 yrs old., better estimate for 15 to 45 yrs

Question 31 (6 marks)

A particle moves in a straight line. At time t seconds, its distance x metres from a fixed point 0 on the line is given by $x = 1 - \cos 2t$.

(a) Sketch the graph of x as a function of t for $0 \leq t \leq \pi$. **2**



Centre = 1
Amplitude = 1
Period = π

(b) Using your graph, or otherwise, to find the times when the particle is at rest and the position of the particle at these times. **2**

When at rest, $v = 0$

$v = \frac{dx}{dt}$ (turning points)

From the graph, $v = 0$ at $x = 0, \pi/2, \pi$

OR

$x = 1 - \cos 2t$

$\frac{dx}{dt} = 2\sin 2t$

$2\sin 2t = 0$

$\sin 2t = 0$

$2t = 0, \pi, 2\pi$

$t = 0, \pi/2, \pi$

Question 31 continues next page

(c) Find the velocity of the particle when $t = \frac{\pi}{4}$. **1**

When $t = \pi/4$

$V = 2\sin 2\left(\frac{\pi}{4}\right)$

$= 2\sin\frac{\pi}{2}$

$= 2\text{m/s}$

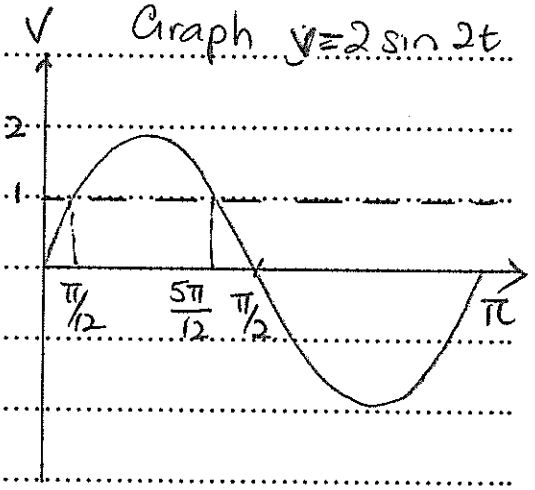
(d) When is the particle's velocity greater than 1 m/s? **1**

$2\sin 2t = 1$

$\sin 2t = \frac{1}{2}$

$2t = \pi/6, 5\pi/6$

$t = \pi/12, 5\pi/12$



From graph $\frac{\pi}{12} < t < \frac{5\pi}{12}$

Question 32 (6 marks)

A function is given by $f(x) = x^4 - 4x^3$.

(a) Find the stationary points, determine their nature, and find any points of inflection. **4**

$f'(x) = 4x^3 - 12x^2$

Stationary points, $f'(x) = 0$

$4x^3 - 12x^2 = 0$

$4x^2(x - 3) = 0$

$x = 0, \quad x = 3$

$f(0) = 0, \quad f(3) = -27$. Stationary points are $(0, 0)$ and $(3, -27)$

Check nature of stationary points:

$f''(x) = 12x^2 - 24x$

when $x = 0$, $f''(0) = 0$ hence $(0, 0)$ is a horizontal point of inflection

when $x = 3$, $f''(3) = 36 > 0$ hence $(3, -27)$ is a minimum point

Inflection points $f''(x) = 0$

$12x(x - 2) = 0$

$x = 0, \quad x = 2$

$x = 0$ is a horizontal point of inflection as shown above

when $x = 2$, $f(2) = -16$

$(2, -16)$ is an inflection point

Question 32 continues next page

(b) Sketch the graph of $f(x) = x^4 - 4x^3$, labelling the stationary points, inflection points and axis intercepts. **2**

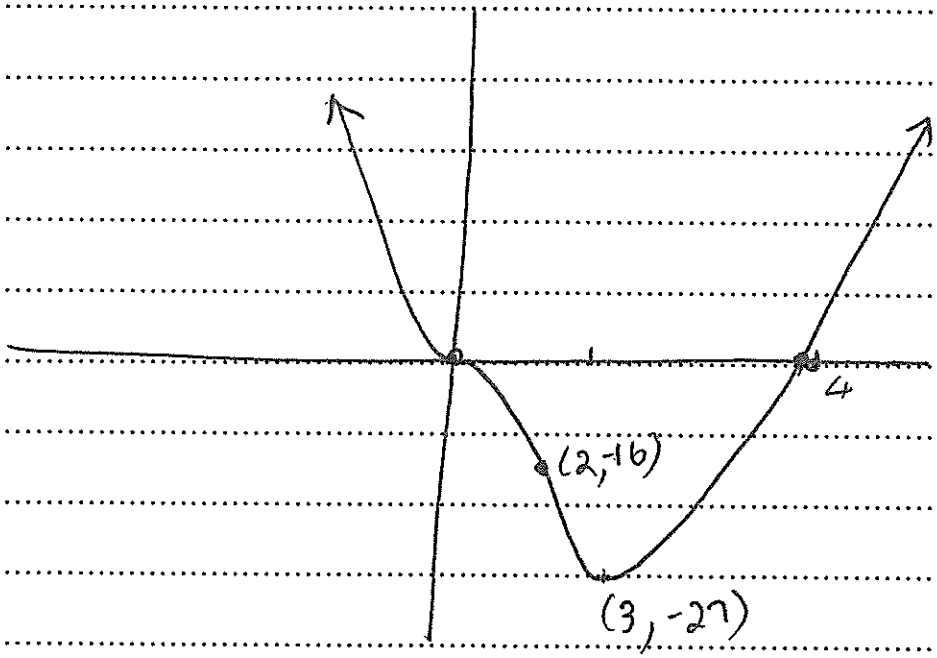
x-intercepts, $f(x) = 0$

$x^4 - 4x^3 = 0$

$x^3(x-4) = 0$

$x = 0, 4$

y-intercept is $y = 0$

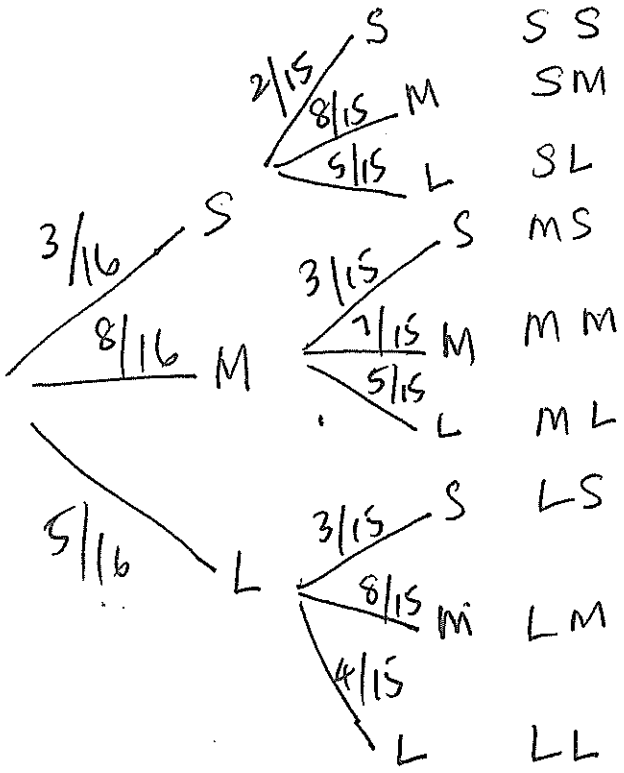


Question 33 (3 marks)

In a laundry basket there are shirts in three sizes, 3 are small, 8 are medium and 5 are large.

Ed takes two shirts from the basket at random.

(a) Draw a tree diagram, clearly showing the outcomes and the correct probabilities on each branch. **2**



(b) Find the probability that Ed chooses two shirts of the same size. **1**

$P(\text{same sizes}) = P(SS) + P(MM) + P(LL)$

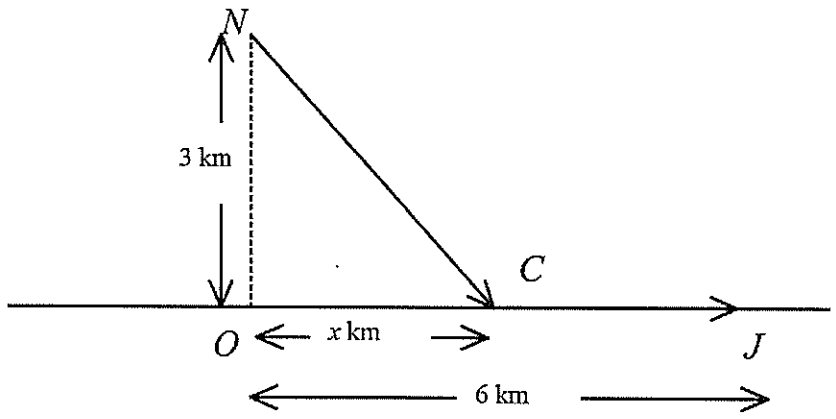
$= \left(\frac{3}{16} \times \frac{2}{15}\right) + \left(\frac{8}{16} \times \frac{7}{15}\right) + \left(\frac{5}{16} \times \frac{4}{15}\right)$

$= \frac{1}{40} + \frac{7}{30} + \frac{1}{12}$

$= \frac{41}{120}$

Question 34 (5 marks)

Nathan is on a paddle board in the ocean 3 km from the nearest point O on a straight beach. Nathan needs to meet his friend Jarrod who is located 6 km along the beach from the point O. Nathan paddles at a rate of 4 km/h to a point C on the beach and then walks at a rate of 5 km/h along the beach to Jarrod. **5**



Show that the total time it takes Nathan to reach Jarrod is given by:

$T(x)=\frac{\sqrt{x^2+9}}{4}+\frac{6-x}{5}$.

Hence, find the minimum time it will take Nathan to reach Jarrod.

Nathan paddles at 4km/h from N to C

Distance from N to C using pythagoras' theorem is $\sqrt{x^2+9}$

Time taken = $\frac{\sqrt{x^2+9}}{4}$

Nathan walks at 5km/h along distance $6-x$

Time taken = $\frac{6-x}{5}$

∴ Total time taken

$T(x) = \frac{\sqrt{x^2+9}}{4} + \frac{6-x}{5}$

Answer space for Question 34 continues next page

To find minimum time, find $T'(x) = 0$

$$T(x) = \frac{1}{4}(x^2+9)^{1/2} + \frac{1}{5}(6-x)$$

$$T'(x) = \frac{1}{2} \times \frac{1}{4}(x^2+9)^{-1/2}(2x) - \frac{1}{5}$$

$$= \frac{x}{4\sqrt{x^2+9}} - \frac{1}{5}$$

$$= \frac{5x - 4\sqrt{x^2+9}}{20\sqrt{x^2+9}}$$

$T'(x) = 0$

$$\frac{5x - 4\sqrt{x^2+9}}{20\sqrt{x^2+9}} = 0$$

$$5x - 4\sqrt{x^2+9} = 0$$

$$4\sqrt{x^2+9} = 5x$$

Square both sides: $16(x^2+9) = 25x^2$

$$9x^2 = 144$$

$x = \pm 4$ but x cannot be negative

$\therefore x = 4$

Check $x = 4$ is a minimum

x	3	4	5
$T'(x)$	-0.02	0	-0.07

$x=4$ minimum

Time taken is when $x = 4$

$$T(4) = \frac{\sqrt{4^2+9}}{4} + \frac{(6-4)}{5}$$

$$= 1.65 \text{ hours}$$

Minimum time is 1 hour 39 mins

END OF EXAMINATION

Section II Extra writing space

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Trial Feedback – Advanced 2020

Q18a) – Done well –

Very few students did errors in subing (2,4) and incorrectly calculating value of C.

Q18 b) Mostly done well. Few did not realise $\frac{dy}{dx} = m$ some did $m_1 m_2 = 1$, MOST did not leave answers as integers in general form.

Q 19) Very hard question. Students who did well (exception A few top kids) forgot $-\frac{\pi}{3}$ as a solution.

Mostly done badly.

Errors – Students find 1 solution at $2x - \frac{\pi}{3} = \frac{\pi}{3}$ and not the rest.

Error – Many made the error $2x - \frac{\pi}{6}$ instead of $2x - \frac{\pi}{3}$

Error – Students do not write the new domain

i.e $\square \leq 2x - \frac{\pi}{3} \leq \square$

Some wrote $0 \leq 2x - \frac{\pi}{3} \leq \frac{11\pi}{3}$

↑ should have been $-\frac{\pi}{3}$

Q20 Not many students remembers that median is $\int f(x) = 0.5$ and you have to find the limit value. Those who remembered did well. Some did $\int_0^{\frac{\pi}{6}} f(x) = 0.5$ and got lost what they were finding.

Q21a) Marked very generously. Must use trazezoidals (not triangles) to find area. Confusion with 'h' value

b) Done extremely badly. This did not require chain rule. It was a logical question. Students

did not correctly convert seconds to hours.
Many did $6 \times 60 = 360$ m/hour
Many tried $V = \frac{1}{3}Ah$

Q22 Some students did $4\int \frac{2x+2}{4x^2+8x+1}\,dx$
$= 4\ln(4x^2+8x+1)+C$
rather than $\frac{4}{4}\int \frac{2x+2}{4x^2+8x+1} = \frac{1}{4}\ln(4x^2+8x+1)+C$

Many forgot (+C). Some students were lost and did not use the reference sheet.

Q23 a) Done well – some rounded to nearest cent incorrectly e.g. 0.392 and/or did not leave answer to 2dp for money. ≈ 0.4

b) Really badly done. Not realising that table had to be used.
– Not seeing that the repayments were larger than borrowed amount
– Not realising it was opposite of part(a).

Q24 a) If realised total probability = 1 then did well
– some had silly errors like $7a = 1$ $a = 72$.

b) Mostly did OK. Some made it harder by $1 - \int_3^6 - \int_5^6$

Q25 a) Given marks for both algebra and graph. Some did not realise $\sin x = \cos x \Rightarrow \tan x = 1$ some had $\tan x = 0$. and some $\sin x - \cos x =$ and could not solve.

Q25 b) Mostly recognised upper - lower. and solved it fine - Some errors not recognising $\sin\frac{5\pi}{4} = -\sqrt{2}$.

Q26) Errors included $\frac{d}{dx} e^{x^3} = 3xe^{x^3}$ or $3e^{x^3}$

Q27) . Many not realise $f(x+h) = (x+h)^2 + 3(x+h)$ they did $x^2 + 3x + h$

- Some Students forgot to write $\lim_{h \to 0}$

Q28 a) Done very badly Not been able to make the drawing correctly. Students should make another 2D ~~shape~~ figure and not draw on the 3D picture

b) Done well. Some could not see that $\cot\theta = \frac{1}{\tan\theta}$

c) Many left ~~b~~ blank. if cosine rule is started correctly 1 mark is given. Many used part (b) for this section and did not see that the triangle is not right angled.

Feedback Yr 12 Adv 2020
Trial

Question 29

part a and b well done
part c many gave 7 weeks as answer.

Question 30

Done well

Question 31

part a done ok.
part b many only found 2 values (1 mark given) for the time
part c done well.
part d badly done, some could find the values of t but didn't realise that the question said greater than 1 m/s. so their answer was $t = \pi/12$ and $t = \frac{5\pi}{12}$.

Question 32

many incorrectly identified the horizontal point of inflection (0,0)
Good with finding the minimum point and point of inflection

Question 33

many left out the outcomes but 2 marks still given.
If there were incorrect probabilities on the tree branches then 1 mark given

PTO →

Question 34

Show part was done well.

Differentation was not done well

If they managed to find $T'(x)$, solving $T'(x) = 0$ was another problem.

Top class did well but other 2 classes did badly.

1 mark - show question

3 marks for differentation and solving for x.

1 mark for the isubstition and finding the time taken